National Open School of Trinidad and Tobago

LESSON SUMMARY

CXC CSEC MATHEMATICS

UNIT Seven:
Geometry

Lesson
13

Properties of Shapes

Textbook: Mathematics, A Complete Course by Raymond Toolsie, Volume 1 and 2.

(Some helpful exercises and page numbers are given throughout the lesson, e.g. Ex 9I page 481)

INTRODUCTION

This lesson builds on concepts established in lesson twelve. It looks at the properties of many figures including polygons and circles. Particular attention is placed on angles formed in a circle.

OBJECTIVES

At the end of this lesson you will be able to:

a) Identify simple plane figures which possess symmetry
b) Use properties of polygons to solve geometric problems
c) Use properties of a circle to solve geometric problems

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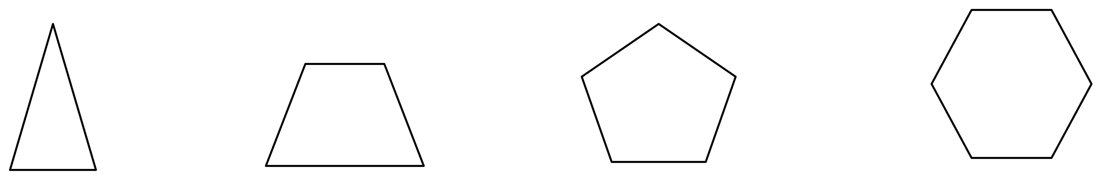
7.2 Polygons

A polygon is a flat (plane) shape with three or more sides formed from straight lines. The point where two sides meet in a polygon is called a vertex.

Examples of polygons are:

Triangles trapeziums pentagons hexagons.



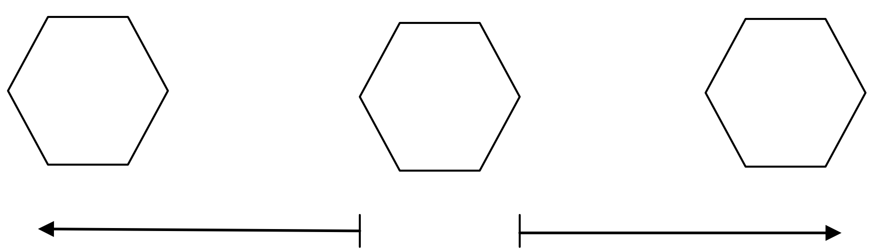
Symmetry

This describes patterns shapes or plane figures may have whereby they look the same if moved or rotated.

Translational Symmetry

If a shape or plane figure is moved along a straight line, without turning and it looks the same then it has translational symmetry. All polygons has translational symmetry.

Example:



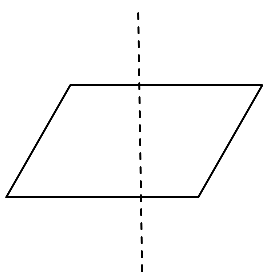
Left Right

Bilateral Symmetry

This is an arrangement of a shape or plane figure along a central axis such that the shape has similar left and right halves.

Example:

Non-example:

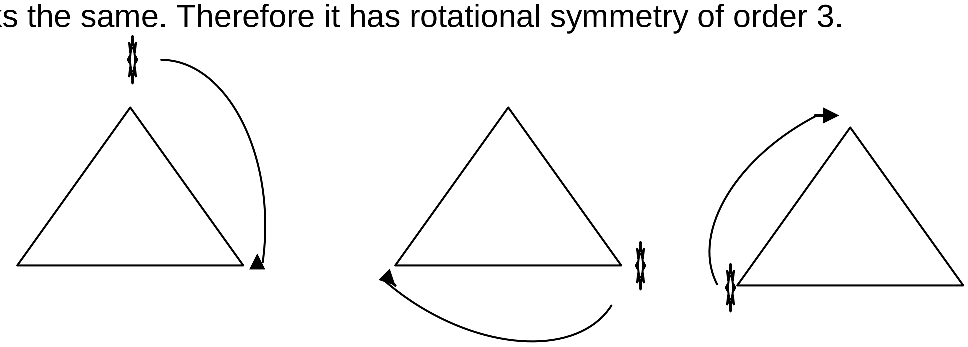


Rotational Symmetry

A shape has rotational symmetry if it looks the same when rotated through a given angle. The number of times this can happen is called the order of the symmetry.

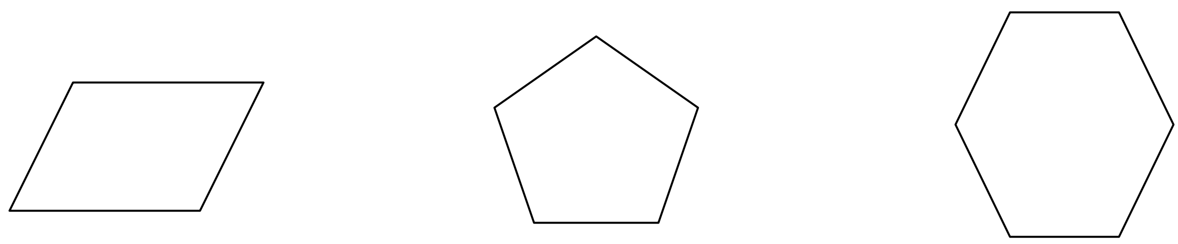
Example:

An equilateral triangle can be rotated three times through an angle of 120^0 and it still looks the same. Therefore it has rotational symmetry of order 3.



ACTIVITY 1

Which of the following shapes possess rotational symmetry when rotated through an angle of 60^0 . State the order of the symmetry.



Angle properties of a polygon

1. The interior angles of any polygon add up to 180^0 (n-2), where n is the number of sides.

2. The exterior angle add up to 360^0.

   Example:

   Determine the sum of the interior angles of a polygon with 18 sides. (Ex 9l page 481)

   Solution: using the formula where n = 18

   $180^0(18-2)$

   $= 180^0\ (16)$

   $= 2880^0$

ACTIVITY 2

A hexagon has interior angles of 95^0 and 175^0. The remaining angles are equal. Calculate the size of each unknown interior angle.

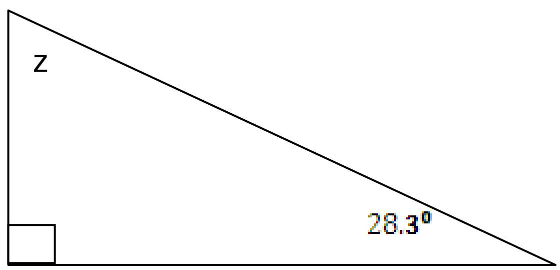
7.3 Triangle

Angle properties of a triangle

The angle properties can be used to solve geometric problems.

i) The interior angles of a triangle add up to 180^0.

Example: Determine the magnitude of angle z, giving a reason for your answer. (Ex 9e page 446)



Solution:

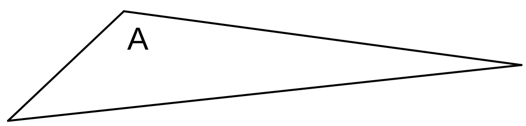
$z + 90^0 + 28.3^0 = 180^0$

$z + 118.3^0 = 180^0$

$z = 180^0 - 118.3^0$

$z = 61.7^0$

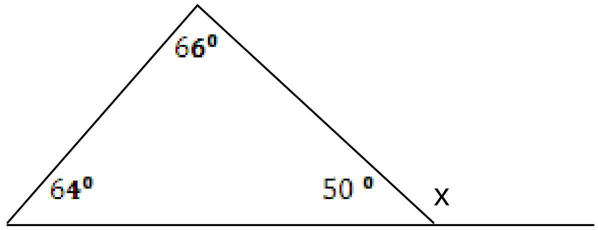
ii) The longest side of a triangle is opposite the largest angle in the triangle and the shortest side is opposite the smallest angle in the triangle.



A is opposite the longest side therefore it is the largest angle in the triangle.

iii) The exterior angle in a triangle is equal to the sum of the two interior opposite angle.

Example: Evaluate angle x



Solution:

According to the property stated above

$x = 66^0 + 64^0$

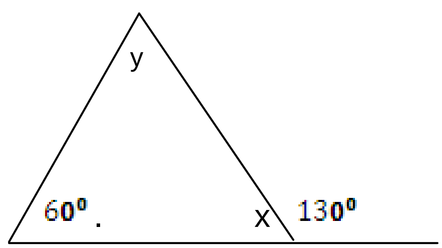
$x = 130^0$.

iv) The sum of three exterior angles add up to 360^0.

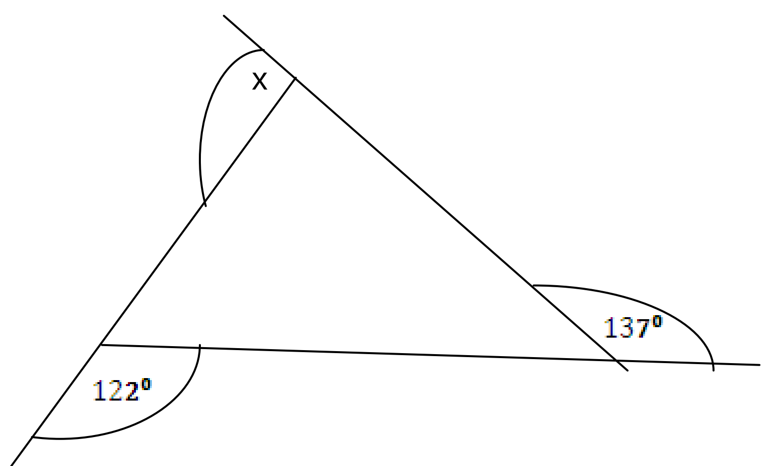
ACTIVITY 3

Evaluate x and y in the following give reasons for your answers.

a.



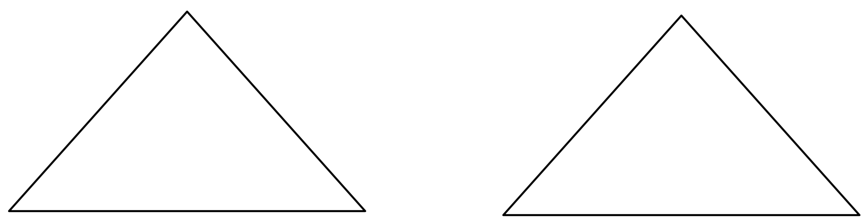
b.



Congruent Triangle

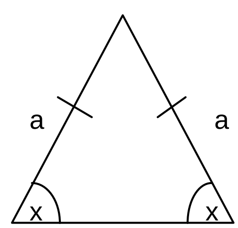
Congruent triangles are triangles that are identical in every way. That is, same size and angles.

Example:



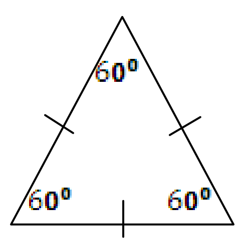
Isosceles Triangles: Two sides and two angles are equal.

Example:



Equilateral Triangles: The angles equal 60^0 and the three sides are equal.

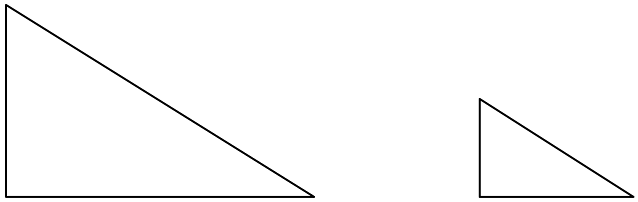
Example:



Similar figures

Similar figures have same angles but different sizes.

Example:



The two triangles are similar. They have the same size angles. The length of one side of the triangle is some number k times the corresponding length of the other triangle. Also the area of one triangle is k^2 by the area of the other.

The area of the first triangle is $24cm^2$ and the lengths of the sides are twice the lengths of the corresponding sides of the second triangle. What is the area of the second triangle?

Solution:

K = 2, therefore

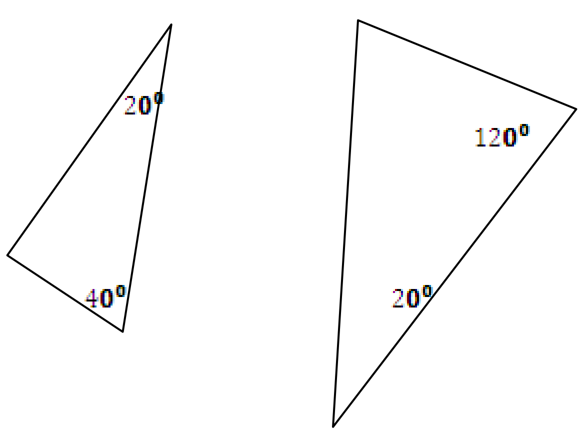
$$the\ area\ of\ the\ second\ triangle \times 2^2 = the\ area\ of\ the\ first\ triangle$$

$$the\ area\ of\ the\ second\ triangle = \frac{24cm^2}{4}$$

$$= 6cm^2$$

ACTIVITY 4

a. Prove that the two triangles are similar. (Ex 9h page 640)



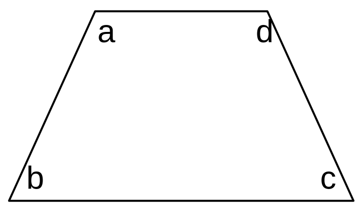
b. If the lengths of the sides of the second triangle is 3 times the lengths of the corresponding sides of the first triangle and the area of the first triangle is $39cm^2$ find the area of the second triangle.

7.4 Quadrilaterals

This is a four sided plane shape or figure. Rectangles, trapeziums and parallelograms are examples of quadrilaterals.

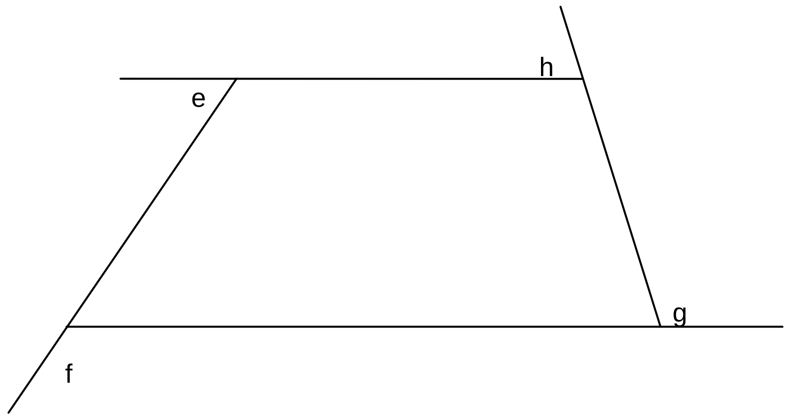
Properties of quadrilaterals

The four interior angles add up to 360^0.



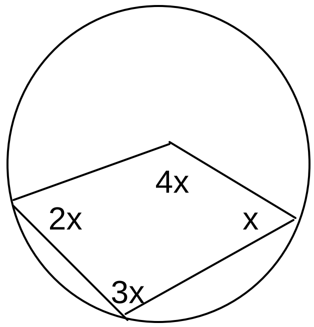
$$a + b + c + d = 360^0$$

The four exterior angles add up to 360^0



$$e + f + g + h = 360^0$$

Example: Calculate the size of angle x. give reasons for your answer. (Ex 9j page 470)



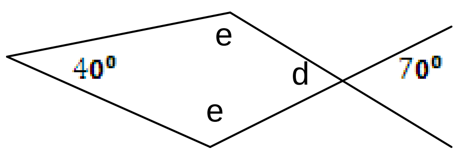
This is a quadrilateral in a circle the same rules apply.

$2x + 3x + x + 4x = 360^0$

$10x = 360^0$

$x = 36^0$

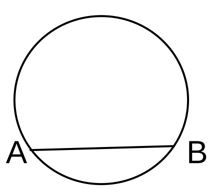
Calculate the size of angles d and e. give reasons for your answers.



Lines, angles and circles

A chord of a circle is a straight line joining any two points on the circumference.

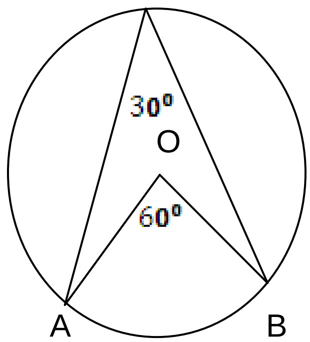
Example:



AB is a chord.

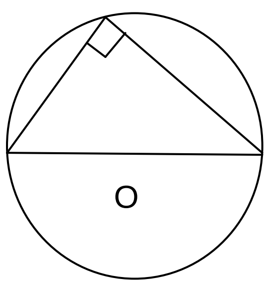
The angle at the centre of a circle is twice the angle at the circumference if the angles stand on the same chord or arc.

Example:



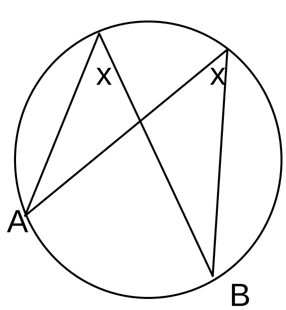
The angle in a semi-circle is a right angle.

Example:



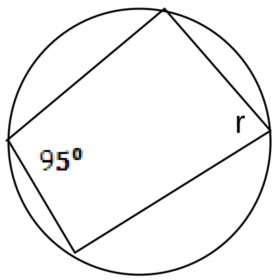
Two angles formed at the circumference and standing on the same arc or chord are equal.

Example:



The opposite angles of a cyclic quadrilateral are supplementary.

Example:



Determine the magnitude of angle r.

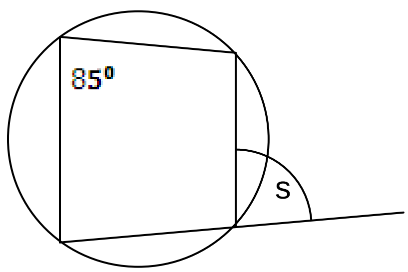
$r + 95^{0} = 180^{0}$

$r = 180^{0} - 95^{0}$

$r = 85^{0}$.

The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

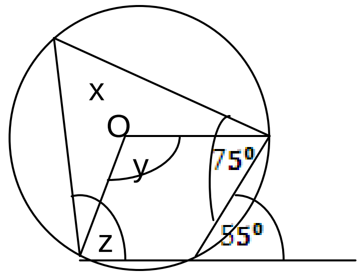
Example: Determine angle s.



s = 85^{0}

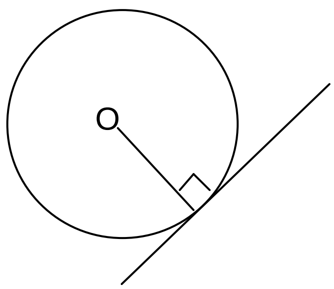
ACTIVITY 5

Without measuring, determine the magnitude of each letter representing an angle in the circle, giving a reason for each of your answer.



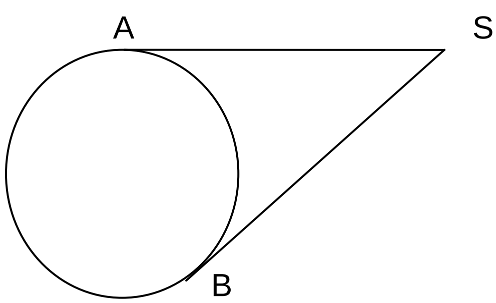
A tangent of a circle is perpendicular to the radius.

Example:



The lengths of two tangents from the circumference to an external point are equal.

Example:

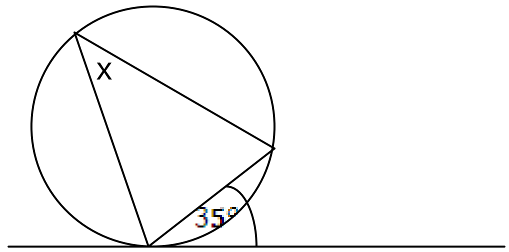


AS = BS

An angle between a tangent of a circle and a chord is equal to the angle in the alternate segment.

Example:

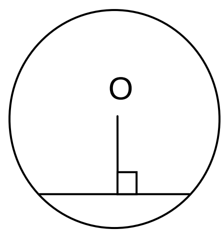
Estimate the value of x in the following:



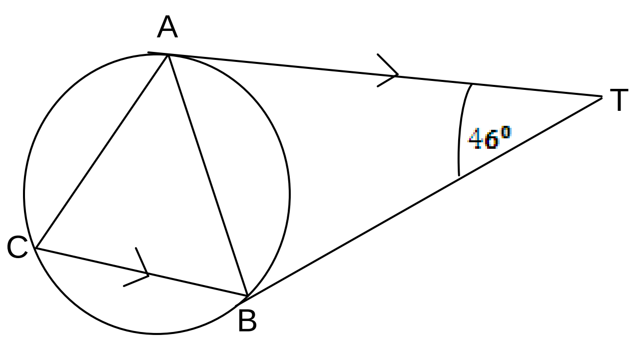
Solution: By the property stated above $x = 35^0$.

The line joining the centre of the circle to the midpoint of a chord is perpendicular to the chord.

Example:



ACTIVITY 6



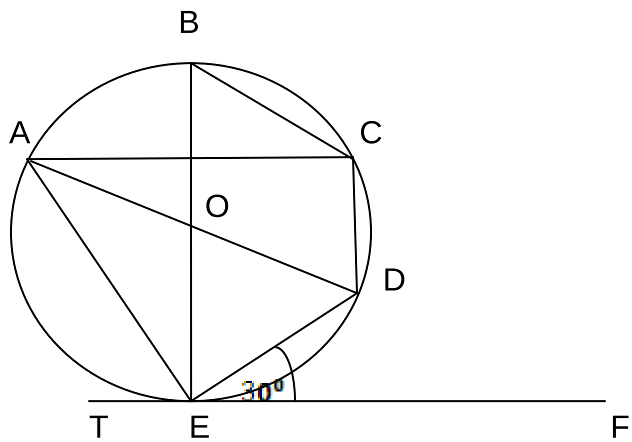
In the figure above AT and BT are tangents to the circle, and CB is a parallel to AT.

If $A\hat{T}B = 46^0$, evaluate:

(a) $A\hat{B}C$ (b) $A\hat{C}B$ (Ex 22d page 1095)

ASSESSMENT

CXC question



In the figure above, **not drawn to scale,** O is the centre of the circle ABCDE and TEF is a tangent to the circle at E.

Given that DEF = 30^0 , calculate, giving reasons to support your answer, the size of the angle

(i) ACD
(ii) EAD
(iii) EOD
(iv) BCD.

Conclusion

The concept of angle properties of polygons and circles were developed in this lesson. These properties were used to solve geometric problems. In the lesson that follows we will look at construction of geometric figures.